BENHA UNIVERSITY

FACULTY OF ENGINEERING
AT SHOUBRA

PHYSICAL AND MATHEMATICAL DEPARTMENT

ENGINEERING MECHANICS

DYNAMICS PROBLEMS

FOR
PREPARATORY YEAR

BY

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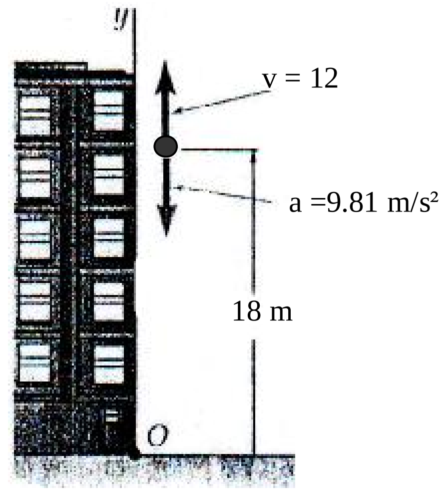
Chapter I- Rectilinear Motion

1) A ball is thrown from the top of a tower **18** m height, with a velocity of **12** m/s vertically up ward, knowing that the acceleration of the ball is constant and equal to **9.81** m/s² down ward, determine

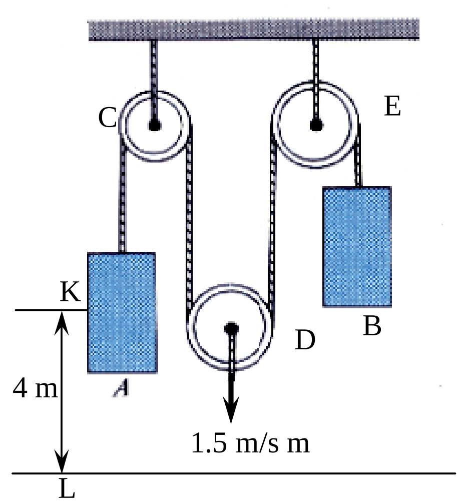
(a) the velocity and elevation y of the ball above the ground at any time (t)

(b) the highest elevation and the corresponding value of (t).

(c) the time when the ball will hit the ground and the corresponding velocity.



2) Two block **A** and **B** are connected by a cord passing over three pulleys C,D, and E as shown. Pulley **C** and **E** are fixed, while **D** is pulled downward with a constant velocity of **1.5** m/s. at **t=0** block (**A**) starts moving downward from (**K**) to (**L**) with a constant acceleration and no initial speed knowing that the velocity of block (**A**) is **6** m/s as it passes through point (**L**), determine the velocity and the acceleration of point(**B**) when (**A**) passes through **L**.



3) a particle moving in straight line with acceleration **$a=-kv^3$,** show that the velocity is

$v = v_0 / (1+kv_0x)$, where v_0 is the initial velocity.

4) A particle initially at rest and from origin, moves from a fixed point in a straight line so that at the end of (t) second , its acceleration

$$\sin t + \frac{1}{(1+t)^2}$$

show that its distance from the fixed point at the end of π seconds is

$$2\pi - \ell n\,(\pi + 1).$$

5) the acceleration of a particle falling through the atmosphere is defined by $a = g(1-k^2v^2)$ knowing that the particle starts at $t=0$ and $x=0$ with no initial velocity (**a**) show that the velocity at any time is $v= 1/ k \tanh (kgt)$

(**b**) write an equation defining the velocity for any value of (x).

(**c**) why $v=1/k$ called the terminal velocity.

6) A particle whose mass is "***m***" is acted by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$ toward the origin. If the particle starts from rest at a distance "***a***". show that it will arrive at the origin in time $\frac{p}{4\sqrt{m}}$

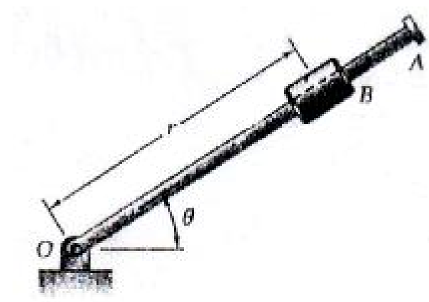
Chapter I- Curvilinear Motion

1) the position of a particle is defined by $\vec{r} = 5\cos 2t\,\vec{i} + 4\sin 2t\,\vec{j}$ where t is the time, determine the magnitude of the velocity and the acceleration of the particle at **t=1** sec, also prove that the path of the particle is **elliptical**.

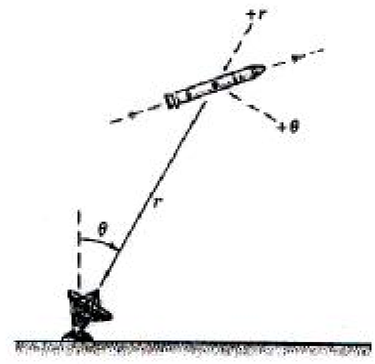
2) if the coordinates of a particle moves in a plane are given by :

$x = e^{\frac{t}{2}}$ and $y = e^{-\frac{t}{2}}$ where **t** is the time, find the equation of the path of the particle and calculate also the velocity and the acceleration of the particle when **t=2** sec.

3) the rotation of the **0.9m** arm **OA** about **O** defined by the relation $\boldsymbol{\theta = 0.15t^2}$, where θ is expressed in radians and **t** in second. Block **B** slides along the arm in such away that its distance from **O** is $\mathbf{r = 0.9 - 0.12\ t^2}$,where **r** expressed in meter and **t** in sec. determine the total velocity and the total acceleration of block **B** after the arm **OA** has rotated through **30º**.



4) A tracking radar lies in the vertical plane of the path of a rocket which is coasting in un powered flight a above the atmosphere. For the instant when **θ = 30º** the tracking data give **r = 80,000 m** , **r° = 1200 m/sec**, **θ° = 0.8 deg/sec**. the acceleration of the rocket is due only to gravitational attraction and for its altitude is **9.20** m/sec² vertically down. For these conditions determine the velocity **V** of the rocket and the values of **r°°** and **θ°°**



5) The plane motion of a particle is given by : $\mathbf{r = 2a \cos\theta}$ **&** $\boldsymbol{\theta = \omega t}$ Find :-

a) the magnitude of the velocity and acceleration.

b) the tangential and normal components of the acceleration.

c) the radius of curvature of the path. where (**$\boldsymbol{\omega}$, a** are constant)

6) the velocities of a particle a long and perpendicular to the radius from a fixed origin are **λr** and **μθ** **find the path** and **show that** the acceleration a long and perpendicular to the radius vector are : $l^2 r - \frac{m^2 q^2}{r}$ and $mq\left(l + \frac{m}{r} \right)$

7) A point describes a circle of radius (a) with uniform speed v; show that the radial and transverse a accelerations are $(-v^2/a)\cos\theta$ and $(-v^2/a)\sin\theta$ if a diameter is taken as initial line and one end of the diameter as pole **$r = 2a\cos\theta$**

8) A particle moves a long a circle **r = 2a cosθ** such that it's a acceleration towards the origin is always zero. Prove that $\frac{d^2q}{dt^2} = -2q^{0}\, cot q$

9) A train is traveling on a curve of radius of curvature **1000** m at the speed of **144** km/h the brakes are suddenly applied, causing the train to slow down at a **constant** rate after **6** sec. the speed has been reduced to **90** km/h. determine the acceleration of the train immediately after the brakes have been applied.

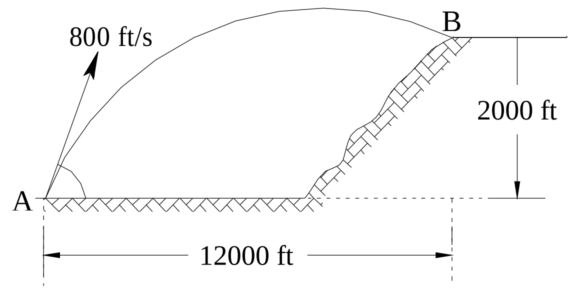
10) A particle moves in a curve (S = C tan ψ), the direction of its acceleration at any point makes equal angles with the tangent and the normal to the path if the speed at ψ=0 be u , show that the velocity and the acceleration at any other point are given by

$$v = ue^{y} \qquad a = \frac{\sqrt{2}u^2 e^{2y}\ cos^2 y}{C}$$

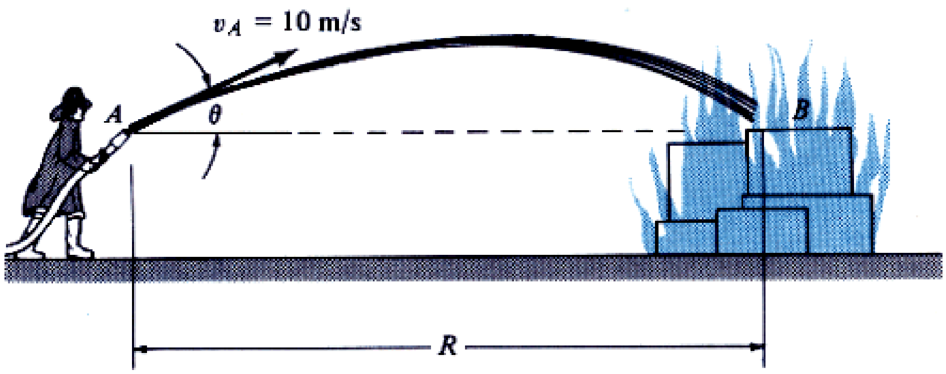
Chapter 2- Projectiles

1) A particle is projected with velocity **u** so that its range on the horizontal plane is **twice** the greatest height. prove that the range is $\mathbf{4u^2/5g}$

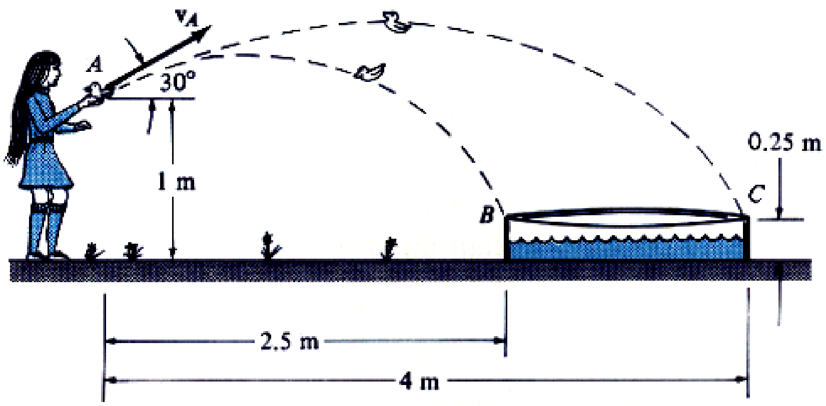
2) A projectile is fired with an initial velocity of **800** ft/s, at a target **B** located **2000** ft above the gun **A** and at a horizontal distance of **12000** ft, determine the value of the firing angle **α**.



3) a fireman at **A** can throw water particle on point **B** at an angle of **15º**. Determine the range **R** if the water has a speed of **10** m/s at **A**.



4) the girl throws the toy at an angle of **30º** from point **A** as shown. Determine the maximum and minimum speed $\mathbf{V_A}$ it can have so that it lands in the pool.



5) a particle is projected from the origin with initial velocity V_0 and angle $\boldsymbol{\alpha}$ with the horizontal, if the particle moves with path equation $y = \frac{1}{\sqrt{3}} x - \frac{1}{1500} x^2$ determine it's range and maximum height. <u>take (g = 10 m/s²)</u>

6) A stone is projected with velocity (**v**) from a height (**h**) to hit a point in the level at a horizontal distance (**R**) from the point of projection, Show that the angle of projection is given by $\mathbf{R^2 \tan^2 \alpha - 2v^2/g \; R \tan \alpha + R^2 - 2hv^2/g = 0}$

Hence deduce that the maximum range on the level for this velocity is

$$R_{max} = \sqrt{\frac{v^4}{g^2} + \frac{2hv^2}{g}}$$

7) A ball is projected so that as just to clear two walls the first of height "**a**" at a distance "**b**" from the point of projection and the second of height "**b**" at a distance "**a**" from the point of projection show that the range on the horizontal plane is **$(a^2 + a b + b^2) / (a + b)$.**

<u>Chapter 3- Simple Harmonic Motion</u>

1) a particle moving in S.H.M, has a velocity of **4** ft/sec. when passing through the center of its path ,and its period is $\boldsymbol{\pi}$ second, what is its velocity when it has described **1** ft from the position in which its velocity is zero?

2) The velocity of a particle moving in S.H.M is **4** m/sec. at a distance of **4** m from the center O, and $\mathbf{2}\sqrt{7}$ m/sec. at a distance of **1** m from **O**. Find the amplitude of the motion and its time period. Also find the velocity at a point of mid way between the extreme position.

3) A point **P** moves in a simple harmonic motion. If the distance of the point **P** from the center of the motion at the ends of three consecutive seconds are **1** cm , **5** cm , **5** cm measured in the same direction with respect to the center, prove that the periodic time of complete oscillation is $\frac{2p}{q}$ where $cos q = \frac{3}{5}$.

4) The velocity of particle moves along x-axis is given by the equation

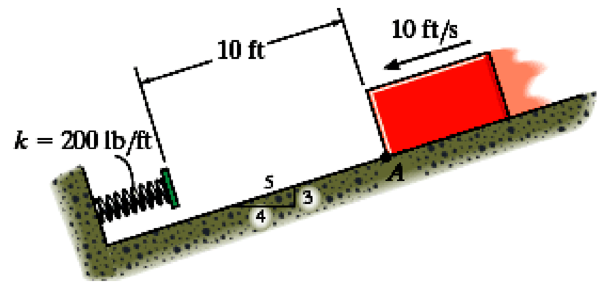
$$\boldsymbol{v^2 = -16x^2+32x+48}$$

prove that the motion is S.H.M, find the center of the motion, amplitude, frequency and max. value of acceleration.

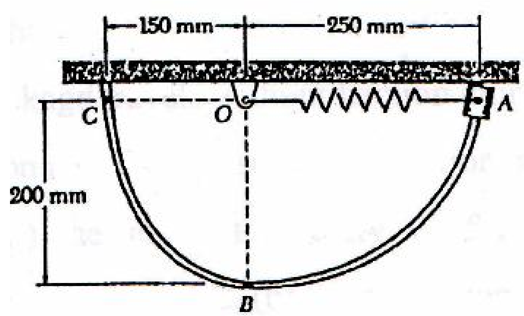
5) One end of alight elastic string of natural length $\mathbf{l}$ and modulus of elasticity **2mg** is attached to a fixed point "O" and the other end to a particle of mass **m**. the particle initially held at rest at "O" is let fall. Show that the particle again reaches to "O" after a time $\sqrt{\frac{2\mathbf{l}}{g}}\left[p + 2 - tan^{-1} 2\right]$.

Chapter 4- Work & Energy

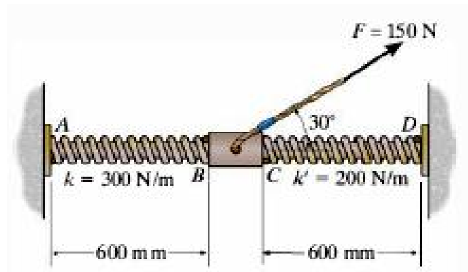
1) The **100**-lb block slides down the inclined plane for which the coefficient of friction is **μ = 0.25**. If it is moving at **10** ft/sec when it reach point **A**, determine the maximum deformation of the spring needed to arrest the motion.



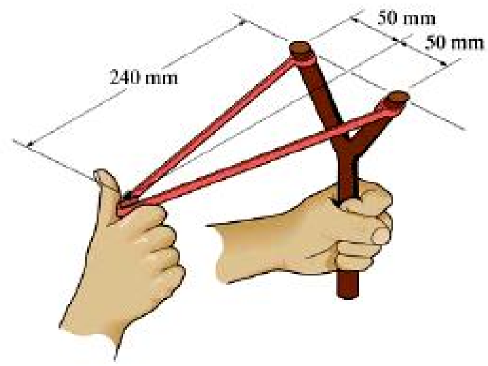
2) A slider of mass **2** kg attached to a spring of stiffness **600** N/m ,the spring is un- deformed when the collar is at **C**, if the collar is released from rest at **A.** Determine the velocity of the slider as it passes through **B** and **C**.



3) Springs **AB** and **CD** have a stiffness of **k = 300** N/m and **k' = 200** N/m, respectively, and both springs have an unstretched length of **600** mm. If the **2**-kg smooth collar starts from **rest** when the springs are unstretched, determine the speed of the collar when it has moved **200** mm.



4) Each of the two elastic rubber bands of the slingshot has an unstretched length of **200** mm. If they are pulled back to the position shown and released from rest, determine the maximum height the **25-**g pellet will reach if it is fired vertically upward. Neglect the mass of the rubber bands and the change in elevation of the pellet while it is constrained by the rubber bands. Each rubber band has a stiffness **k = 50** N/m.



5) Determine whether or not the given force $\boldsymbol{F = (6x^2 + 2xy^2)\, i + (2x^2y + 5)\, j}$ is conservative , if so find its potential function, then calculate the work done by this force along

(i) the line segment from (0,0) to (1,3)

(ii) the part of the parabola $\boldsymbol{y = 3x^2}$ from (0,0) to (1,3)

6) Determine whether or not the given force $\boldsymbol{F} = (3x^2 + y^2)\,\boldsymbol{i} + (2xy)\,\boldsymbol{j} - 3z^2\,\boldsymbol{k}$ is conservative, if so find its potential function, then calculate the work done by this force along the curve $r(t) = (t, 2t, 3t)$ from (0,0,0) to (1,2,3).

7) show that the force $\boldsymbol{F} = (yz)\,\boldsymbol{i} + (xz)\,\boldsymbol{j} + (xy)\,\boldsymbol{k}$ is conservative, then find its potential function, and calculate the work done by this force along the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3} = t,\ 0 \le t \le 1$.

8) A body moves from point (**1,1**) to point (**2,4**) along the path ***y=2x-3***, under the force ***F= (6x+ y) i + (x+2y) j ,*** Find the velocity at point (**2,4**) if the mass of the body is **2-kg** and starts from rest.

9) Find the conservative force which has the potential function $\boldsymbol{u_{(x,y,z)} = -(x^2y + xz^3) + c}$; then find the work done by this force from point (1,-2,1) to point (3,1,4).

Chapter 5- Motion along smooth vertical circle

1) A particle attached to the end of a string of length $(\mathbf{l})$, the upper end is fixed. If the particle is projected horizontally with velocity of $\sqrt{ng\mathbf{l}}$, show that the string becomes slack at a height $\left(\frac{n+1}{3}\right)\mathbf{l}$.

2) A particle of weight (W) is attached to the end of non elastic string of length $(\mathbf{l})$, the upper end is fixed. If the particle do a complete vertical revolution about center "O" and the tensions in the string are mW and nW at the upward and downward positions respectively. Prove that $n = m + 6$ .where m,n are constants.

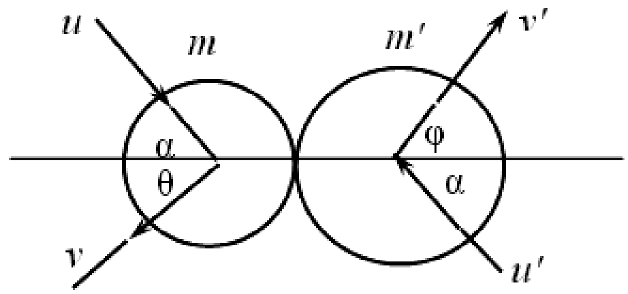
3) A particle moves down outside the surface of smooth vertical circular disc of radius (r) if the particle start from rest at a depth $\frac{r}{2}$ below the highest point, show that it leaves the disc at a height $\frac{r}{3}$ above the center.

4) A particle is projected from the lowest point inside a smooth sphere of radius (r) with velocity sufficient to reach the particle to height $h = \frac{\sqrt{3}}{2} r$ above the center. find the point at which the particle will leave the sphere and show that it will passes through the center.

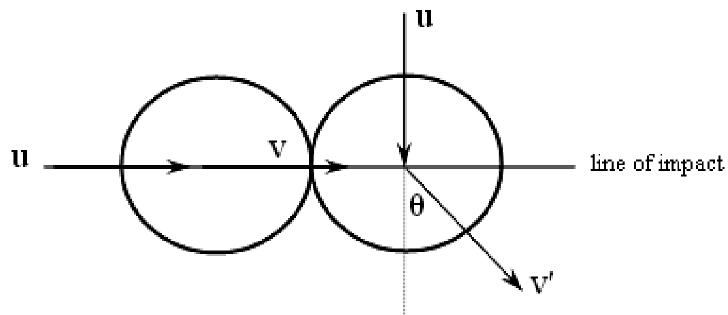
5) A heavy particle of mass (**m**) makes complete revolutions in a smooth circular tube fixed in a vertical plane. Its max. speed is (**n**) times its min. prove that the pressure in the tube when the particle is moving vertically is **$2mg\,(n^2 + 1)/(n^2 - 1)$.**

Chapter 6- Impulse & Momentum (Impact)

1-Two balls of elasticity e moving with equal momentum in opposite parallel1 directions and impinge, prove that they will move after impact in parallel directions with equal momentum.

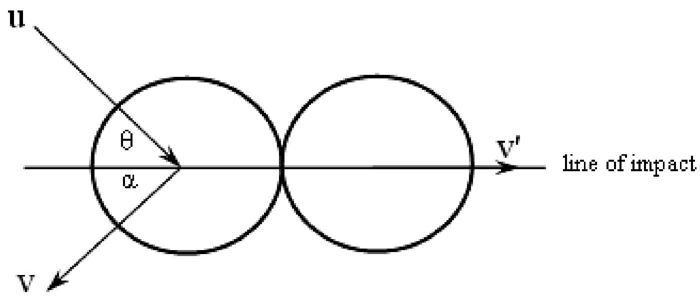


2- A ball impinges on another equal ball moving with the same speed in a direction perpendicular to its own , the line joining the centers of the ball at the instant of impact being perpendicular to the direction of motion of the second ball , if e be the coefficient of restitution , show that the direction of motion of second ball is turned through an angle $\tan^{-1}[(1+e)/2)]$



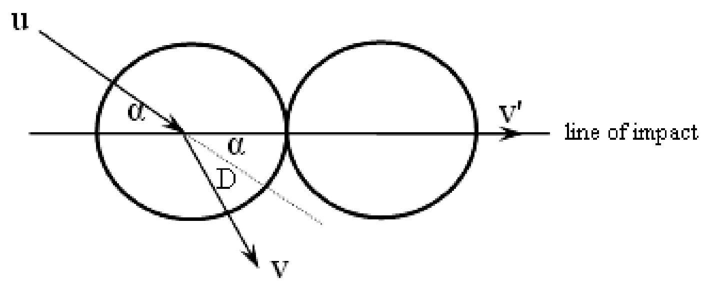
3- A smooth sphere of mass **m** traveling with velocity **u** impinges on another smooth sphere of mass **M** at rest , its original line of motion making an angle **θ** with the line of centers at the moment of impact. Show that the sphere of mass **m** will be deflected through a right angle if

$$tan^2 q = \frac{eM - m}{M + m}$$

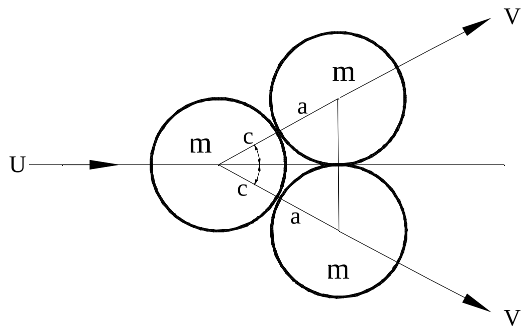


4-A smooth ball impinges on another smooth equal ball at rest in a direction that makes an angle **α** with the line of centers at the moment of impact. Prove that if **D** be the angle through which the direction of the impinging ball is deviated then

$$\tan D = \frac{(1+e)\tan a}{1-e+2\tan^2 a}$$



5- Two equal balls of radius (**a**) are in contact and , a third ball (**m'**) of radius (**c**) moving in the direction of their common tangent impinges on them , prove that the impinging ball will be reduced to rest if $\mathbf{2e = c^2 (a+c)^2 / a^3(2a+c)}$, where **e** is the coefficient of elasticity and the masses of balls are proportional to a^3 and c^3.



6- A ball weighing **10** pound and moving with a velocity **30** ft/s impinges on a smooth fixed plane in a direction making **60°** with the plane, find its velocity and direction of motion after impact if the coefficient of restitution is **2/3**. Find also the loss of kinetic energy.